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Free Vibration Analysis of Euler-Bernoulli Beam with Double Cracks

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Abstract

In this paper, the influence of two open cracks on the dynamic behavior of a double cracked simply supported beam is investigated both analytically and experimentally. The equation of motion is derived by using the Hamilton's principle and analyzed by numerical method. The simply supported beam is modeled by the Euler-Bernoulli beam theory. The crack sections are represented by a local flexibility matrix connecting three undamaged beam segments. The influences of the crack depth and the position of each crack on the vibration mode and the natural frequencies of a simply supported beam are analytically clarified for the single and double cracked simply supported beam. The theoretical results are also validated by a comparison with experimental measurements.

Keywords: Free vibration; Euler-bernoulli beam; Double crack; Eigenvalue; Flexibility matrix

1. Introduction

The dynamic behavior of beam with the cracks is considerably important in many structural designs. When a structure is subjected to damage its dynamic response is varied due to the change of its mechanical characteristics. Chondros and Dimarogonas (1989, 1998) studied the effect of the crack depth on the dynamic behavior of a cantilevered beam. They showed that the increase of the crack depth reduces the natural frequency of the beam. Also, they used energy method and a continuous cracked beam theory for analyzing the transverse vibration of cracked beam. Dado and Abuzeid (2003) studied the modeling and analysis algorithm for cracked Euler-Bernoulli beam by considering the coupling between the bending and axial modes of vibration. This algorithm is applied to the analysis of the vibration behavior of the cracked beam and particularly its natural frequency and mode shapes under the effect of added mass and rotary inertia at the free end. Lin (2004) investigated the direct and inverse methods on the free vibration analysis of the simply supported beams with a crack. The method is based on modeling the beam by Timoshenko beam theory and presents the crack as a mass-less rotational spring. Liu et al. (2003) examined the suitability of using coupled responses to detect damage in thin-walled tubular structures. By coupled response they referred to the ability of a structural member with a circumferential crack to describe composite vibration modes (axial and bending) when excited purely laterally. Zheng and Fan (2003, 2003) studied the stability of a cracked Timoshenko beam column by modified Fourier series. Also, they present simple tools for the vibration and stability analysis of cracked hollow-sectional beams. Maiti et al. (2004) have shown the results of study on crack detection in pipes filled with fluid by the theoretical analysis and the experiment. Recently, Yoon and Son (2004) investigated the effects of the open crack and the moving mass on the

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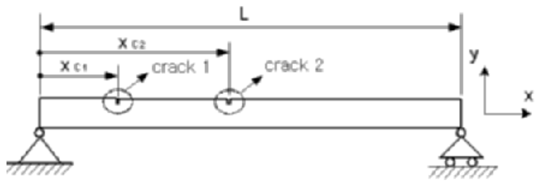
dynamic behavior of the simply supported pipe conveying fluid. They studied about the influences of the crack, the moving mass and its velocity, the velocity of fluid, and the coupling of these factors on the dynamic behavior of the Timoshenko beam. Most of the studies for crack detection were concentrated on the analysis of the effect of a single crack on the dynamics of a simple structure such as a shaft and a beam. In practice, it is sufficiently possible for the two or more cracks to exits on the structures. The dynamic behavior of a double-cracked beam and a rotor with two cracks were investigated by Ruotolo et al. (1996) and Shekar (1999), respectively. Lin et al. (2002) studied beam vibrations with an arbitrary number of double-side crack and single-side crack using the transfer matrix methods. Ostachowicz and Krawczuk (1991) investigated the influence of the position and the depth of two open cracks upon the fundamental frequency of the natural flexural vibrations of a cantilever beam. To model the effect of the local stress in the crack, they introduced two different functions according to the symmetry of the crack. Shen and Pierre (1990) considered the same problem in case of the symmetric cracks. An equation of the bending motion for Euler-Bernoulli beam containing pairs of the symmetrical open cracks was derived by Christides and Barr (1984). The cracks were considered to be normal to the beam's neutral axis and symmetrical about the plane of bending. Douka et al. (2004) studied about a method for determining the location and the depth of cracks in double-cracked beams. Li (2002) investigated the free vibration analysis of a non-uniform beam with an arbitrary number of cracks and concentrated masses.

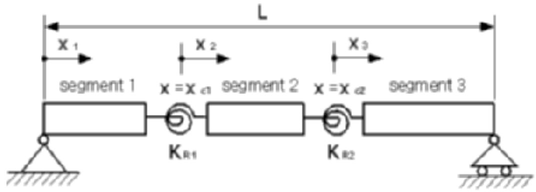
In this study, the effects of each crack on the natural frequency of a simply supported double-cracked beam are investigated. That is, the influences of the crack depth and position of each crack were studied on the dynamic behavior of the simply supported beam system. The theoretical results are also validated by a comparison with experimental measurements in this study. The simply supported beam is modeled by the Euler-Bernoulli beam theory. The cracks are assumed to be always open during vibrations.

2. Mathematical model

The simply supported beam with double cracks is shown in Fig. 1, where L is the total length of the beam,



(a) Double cracks



(b) Beam segments connected by rotational springs

Fig. 1. Geometry of a simply supported double-cracked beam.

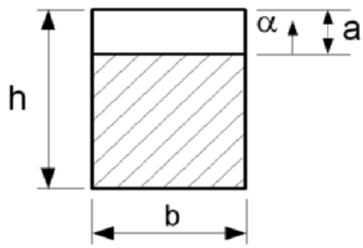


Fig. 2. Geometry of the cracked section of the beam.

x_{c1} and x_{c2} are the positions of each crack from the left-hand hinged end of the beam. In addi-tion, K_{R1} and K_{R2} are the bending stiffness of the first and second cracks, respectively. Figure 2 show the cross-section of the cracked section, where a is the maximum crack depth, b and h are the rectangular cross-section dimensions. Three equations of motion are derived respectively for the three parts of the beam separated by the cracked sections.

2.1 Crack modeling

Consider the bending vibrations of a uniform Euler-Bernoulli beam in the $x-y$ plane, which is assumed to be a plane of symmetry for any cross-section. The crack is assumed to be always open. The additional strain energy due to the crack can be considered in the form of a flexibility coefficient expressed in terms of the stress intensity factor, which can be derived by Castigliano's theorem in the linear elastic range. Therefore the local flexibility (Dimarogonas, 1996) in the presence of the width b of a crack is defined by

$$C_{ijn} = \frac{\partial u_{ijn}}{\partial P_j} = \frac{\partial^2}{\partial P_i \partial P_j}\left(\int_0^b \int_0^{a_n} J(\alpha_n)\ d\alpha dz\right) \quad (n=1,2) \tag{1}$$

where u_{ijn} and α_n are the additional displacement due to a crack and the crack depth as shown in Fig. 2, respectively. P_i is the load in the same direction as the displacement and n (=1,2) represents the first crack and second crack, respectively. The strain energy density function $J(\alpha_n)$ is

$$J(\alpha_n) = \frac{1}{E^*}\left[\left(\sum_{i=1}^{6} K_{Iin}\right)^2 + \left(\sum_{i=1}^{6} K_{IIin}\right)^2 + (1-\nu_p)\left(\sum_{i=1}^{6} K_{IIIin}\right)^2\right] \tag{2}$$

where $E^* = E/(1-\nu_p^2)$ for the plane strain and ν_p is Poisson's ratio. K_I, K_{II} and K_{III} are the stress intensity factors of the fracture mode for the opening, in-plane shear and our-of-shear mode, respectively.

In Eq. (1), the matrix of local flexibility of a beam size depends on the number of the degrees of freedom being considered for the forces and moments of the coordinate system, the maximum being 6×6. In this paper, we considered only the bending vibration because the effect on dynamic behavior of the cracked beam of the local axial and coupled axial and bending is very small (Bamnios et al., 2001; Dado and Abuzeid, 2003). Therefore Eq. (2) can be rewritten as

$$J(\alpha_n) = \frac{1}{E^*}(K_{IMn})^2 \tag{3}$$

The stress intensity factor for the fracture mode I due to moment M is given by

$$K_{IMn} = \frac{6M}{bh^2}\sqrt{\pi\alpha_n}\ F_{II}\left(\frac{\alpha_n}{h}\right) \tag{4}$$

where

$$F_{II}\left(\frac{\alpha_n}{h}\right) = \sqrt{\frac{\tan(\varsigma_n)}{\varsigma_n}}\ \frac{\left[0.923 + 0.199\left(1-\sin(\varsigma_n)\right)^4\right]}{\cos(\varsigma_n)} \tag{5}$$

where $\varsigma_n = \dfrac{\pi\alpha_n}{2h}$. Therefore the relation of local flexibility and the bending stiffness for the beam due to the crack has the following form:

$$K_{Rn} = C_n^{-1} = \left[\frac{\partial^2}{\partial M^2}\int_0^b \int_0^{a_n} J(\alpha_n)\, d\alpha dz\right]^{-1} \tag{6}$$

2.2 Energy of simply supported beam with double cracks

Using the assumed mode method, the transverse displacement of a simply supported double-cracked beam can be assumed to be

$$y_k(x,t) = \sum_{i=1}^{\mu} \phi_{ik}(x)\, q_i(t) \tag{7}$$

where $q_i(t)\left(= e^{j\omega t}, j=\sqrt{-1}\right)$ are generalized coordinates which is time dependent, μ is the total number of the generalized coordinates, and $\phi_{ik}(x)$ are spatial mode functions of the segment k of a simply supported beam. In addition, $k(=1,2,3)$ denotes the number of the segments, which are the parts of the beam separated by the cracked sections. $\phi_{ik}(x)$ can be described as follows:

I) segment 1 ($0 \le x \le x_{c1}$) ;

$$\begin{aligned}\phi_{i1}(x) = A_1 \cos(\lambda_i x) + A_2 \sin(\lambda_i x)\\ + A_3 \cosh(\lambda_i x) + A_4 \sinh(\lambda_i x)\end{aligned} \tag{8}$$

II) segment 2 ($x_{c1} \le x \le x_{c2}$) ;

$$\begin{aligned}\phi_{i2}(x) = A_5 \cos(\lambda_i x) + A_6 \sin(\lambda_i x)\\ + A_7 \cosh(\lambda_i x) + A_8 \sinh(\lambda_i x)\end{aligned} \tag{9}$$

III) segment 3 ($x_{c2} \le x \le L$) ;

$$\begin{aligned}\phi_{i3}(x) = A_9 \cos(\lambda_i x) + A_{10} \sin(\lambda_i x)\\ + A_{11} \cosh(\lambda_i x) + A_{12} \sinh(\lambda_i x)\end{aligned} \tag{10}$$

where λ_i is the frequency parameter, which is easily calculated by using the frequency equation of a simply supported beam(Inman, 1994).

In Eqs. (8)~(10), the constants $A_1, A_2, \cdots, A_{12}$ can be found from the boundary conditions. The boundary conditions of a simply supported beam are

$$\begin{aligned}&\text{at} \quad x=0, \quad \phi_{i1}(0)=0 \quad \text{and} \quad \frac{\partial^2 \phi_{i1}(0)}{\partial x^2}=0,\\ &\text{at} \quad x=L, \quad \phi_{i3}(L)=0 \quad \text{and} \quad \frac{\partial^2 \phi_{i3}(L)}{\partial x^2}=0\end{aligned} \tag{11}$$

The boundary conditions for the transverse deflection, bending moment, shear force and slope at the each crack are

$$\phi_{i1}(x_{c1}) = \phi_{i2}(x_{c1}), \quad \frac{\partial^2\phi_{i1}(x_{c1})}{\partial x^2} = \frac{\partial^2\phi_{i2}(x_{c1})}{\partial x^2},$$
$$\frac{\partial^3\phi_{i1}(x_{c1})}{\partial x^3} = \frac{\partial^3\phi_{i2}(x_{c1})}{\partial x^3}, \tag{12}$$
$$\frac{\partial\phi_{i2}(x_{c1})}{\partial x} - \frac{\partial\phi_{i1}(x_{c1})}{\partial x} = \frac{EI}{K_{R1}}\frac{\partial^2\phi_{i2}(x_{c1})}{\partial x^2}$$

$$\phi_{i2}(x_{c2}) = \phi_{i3}(x_{c2}), \quad \frac{\partial^2\phi_{i2}(x_{c2})}{\partial x^2} = \frac{\partial^2\phi_{i3}(x_{c2})}{\partial x^2},$$
$$\frac{\partial^3\phi_{i2}(x_{c2})}{\partial x^3} = \frac{\partial^3\phi_{i3}(x_{c2})}{\partial x^3}, \tag{13}$$
$$\frac{\partial\phi_{i3}(x_{c2})}{\partial x} - \frac{\partial\phi_{i2}(x_{c2})}{\partial x} = \frac{EI}{K_{R2}}\frac{\partial^2\phi_{i3}(x_{c2})}{\partial x^2}$$

where K_{R1}, K_{R2} are the bending stiffness of the first and second cracks in Eq. (6), respectively. E is the modulus of elasticity of the beam and I is the moment of inertia of the beam cross-section. In Fig. 1, the kinetic and potential energy of the simply supported beam with the double cracks can be written as

$$T_p = \frac{1}{2}m\sum_{i=1}^{\mu}\left[\int_0^{x_{c1}}\{\dot{y}_{i1}(x,t)\}^2 dx + \int_{x_{c1}}^{x_{c2}}\{\dot{y}_{i2}(x,t)\}^2 dx + \int_{x_{c2}}^{L}\{\dot{y}_{i3}(x,t)\}^2 dx\right] \tag{14}$$

$$V_p = \frac{1}{2}EI\sum_{i=1}^{\mu}\left[\int_0^{x_{c1}}\{y_{i1}''(x,t)\}^2 dx + \int_{x_{c1}}^{x_{c2}}\{y_{i2}''(x,t)\}^2 dx + \int_{x_{c2}}^{L}\{y_{i3}''(x,t)\}^2 dx\right]$$
$$+\frac{1}{2}\left\{K_{R1}\left(\frac{y_{i2}(x_{c1})}{\partial x} - \frac{y_{i1}(x_{c1})}{\partial x}\right)^2 + K_{R2}\left(\frac{y_{i3}(x_{c2})}{\partial x} - \frac{y_{i2}(x_{c2})}{\partial x}\right)^2\right. \tag{15}$$

where $(\cdot)$ denotes $\partial/\partial t$, $(')$ represents $\partial/\partial x$ and m is the mass per unit length of the beam.

2.3 Equation of motion

The equation of motion of a simply supported double-cracked beam is obtained by the application the Hamilton's principle. The equation of motion can be matrix form as follow:

$$\mathbf{M}\ddot{\mathbf{q}} + \mathbf{K}\mathbf{q} = \mathbf{0} \tag{16}$$

where the matrices $\mathbf{M}$ and $\mathbf{K}$ are

$$\mathbf{M} = \sum_{i=1}^{\mu} m\left[\int_0^{x_{c1}}\phi_{i1}(x)\phi_{j1}(x)dx + \int_{x_{c1}}^{x_{c2}}\phi_{i2}(x)\phi_{j2}(x)dx + \int_{x_{c2}}^{L}\phi_{i3}(x)\phi_{j3}(x)dx\right] \tag{17}$$

$$\mathbf{K} = \sum_{i=1}^{\mu} EI\left[\int_0^{x_{c1}}\phi_{i1}(x)\phi_{j1}''''(x)dx + \int_{x_{c1}}^{x_{c2}}\phi_{i2}(x)\phi_{j2}''''(x)dx + \int_{x_{c2}}^{L}\phi_{i3}(x)\phi_{j3}''''(x)dx\right] \tag{18}$$

By inserting boundary conditions Eqs.(11)~(13) into the Eqs. (8)~(10), 12 by 12 matrix equation is obtained. The natural frequencies of the double-cracked beam can be obtained by imposing zero value on the determinant of this coefficient matrix.

3. Experiments

In order to validate the model presented in this study, natural frequencies of a simply supported beam with the double cracks are measured by experiments. The length of the beam is 0.4 m and the beam used in the experiments had square cross-section, 0.01×0.01 m^2. The material properties are: the modulus of elasticity of the beam E=2.16×10^{11} N/m^2 and density ρ=7650 kg/m^3. In addition, the cracks were modeled by sawing cuts. The set-up of the experiment of a simply supported beam with double cracks is shown in Fig. 3. One of the hinges is supported the bolts with sharp tip. The other hinge is supported using the square zig to realize the axially movable end. It is evident that both ends are fixed to have no translation motion but are free to rotate and nave no bending moments. An impact test was used by an impact hammer (DYTRAN, series 5801 A), an accelerometer (B&K, type 4507) and a dynamic signal analyzer (LMS, Cada-x 3.5D).

4. Results and discussion

In this study, the effects of the double cracks on

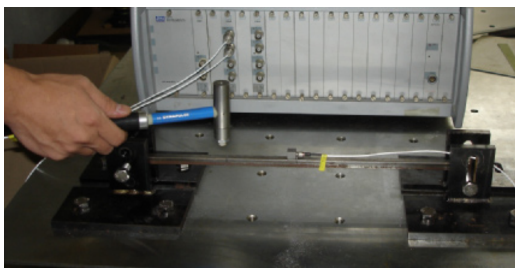


Fig. 3. Experiment set-up of a simply supported beam with the double cracks.

natural frequency of a simply supported beam were investigated by the comparison between the theoretical results and experimental measurements. The equation of motion is computed by the forth order Runge-Kutta method. We have studied the natural frequencies of a simply supported double-cracked beam for the first and second modes of vibration. For simplicity, the following dimensionless quantities are introduced:

$$\text{for single crack}: \quad \xi_c = \frac{x_c}{L}, \quad H = \frac{a}{h} \qquad (19)$$

$$\text{for double cracks}: \quad \xi_{c1,2} = \frac{x_{c1,2}}{L}, \quad H_{1,2} = \frac{a_{1,2}}{h}$$

First of all, the accuracy of the present numerical results needs to be confirmed. Tables 1 and 2 represent the natural frequency of a simply supported beam with a single crack for the first mode and second mode of vibration, respectively. In these Tables, comparison between theoretical results and experimental measurements is also given. In Table 1, the maximum difference between the two results is less than 1.25%. It can be found that the theoretical results are a good agreement with the experimental measurements. In Table 2, natural frequency of the second mode of vibration, the two results are found to be almost identical. But, when the crack position is 0.3 and the crack depth is 0.5, the difference between the two results is 3.73 %. Totally, when the crack depth is 0.5, the difference between the theoretical results and experimental measurements is largest.

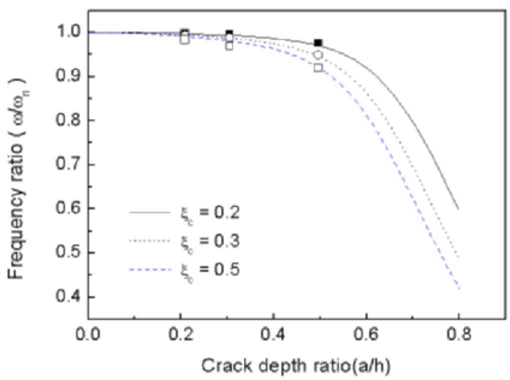
Figure 4 shows the ratio of natural frequency of a simply supported single cracked beam for the first and second modes of vibration. In figures, the axis of the ordinates is the frequency ratio, where ω and ω_n are the natural frequency of a cracked beam and the natural frequency of an un-cracked beam, respect-

Table 1. Natural frequency of a simply supported beam with a crack for the first mode.

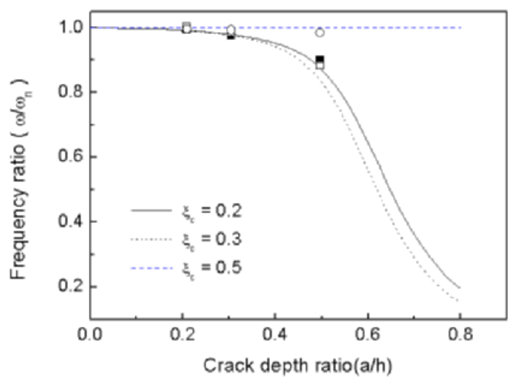
Crack position ($\xi_c = x_c / L$)	Crack depth ($H = a/h$)	Natural frequency [Hz] T:Theory	Natural frequency [Hz] E:Experiment	Error(%) :|E-T|/E×100
Un-cracked beam		150.59	151.5	0.60
0.2	0.1	150.48	151.2	0.48
	0.3	149.61	151.0	0.92
	0.5	145.95	147.8	1.25
0.3	0.1	150.38	150.5	0.08
	0.3	148.75	149.8	0.70
	0.5	142.16	143.8	1.14
0.5	0.1	150.27	149.2	0.72
	0.3	147.81	146.8	0.69
	0.5	138.32	139.2	0.64

Table 2. Natural frequency of a simply supported beam with a crack for the second mode.

Crack position ($\xi_c = x_c / L$)	Crack depth ($H = a/h$)	Natural frequency [Hz] T:Theory	Natural frequency [Hz] E:Experiment	Error(%) :|E-T|/E×100
Un-cracked beam		602.37	602.5	0.02
0.2	0.1	601.18	599.8	0.22
	0.3	590.21	588.2	0.34
	0.5	556.08	568.5	2.18
0.3	0.1	601.16	604.2	0.50
	0.3	588.50	594.2	0.95
	0.5	543.42	564.5	3.73
0.5	0.1	602.37	599.8	0.43
	0.3	602.37	597.8	0.76
	0.5	602.37	593.0	1.58

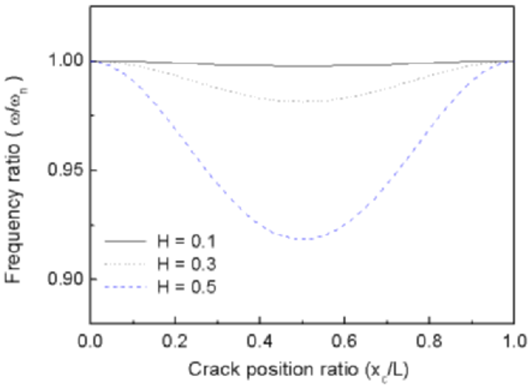


(a) First mode

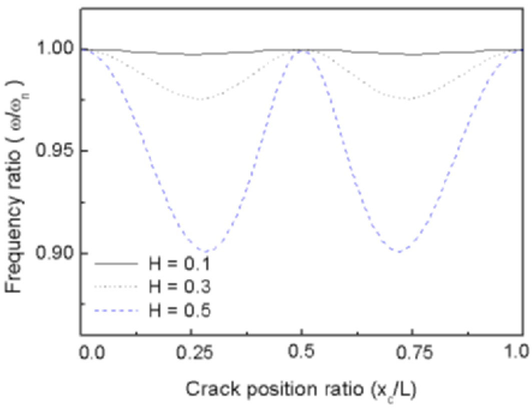


(b) Second mode

Fig. 4. Natural frequency of a simply supported beam according to the crack depth ; experiment, ξ_c =0.2, ■; ξ_c =0.3, □; ξ_c =0.5 ○.

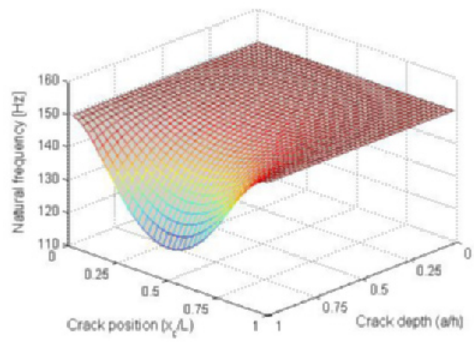


(a) First mode

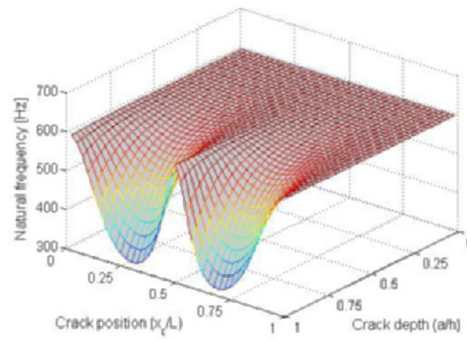


(b) Second mode

Fig. 5 Natural frequency of a simply supported beam according to the crack position.



(a) First mode



(b) Second mode

Fig. 6. Contours of natural frequency of a simply supported beam due to the crack.

ively. These figures show that the theoretical results are a good agreement with the experimental measurements. In Fig. 4 (a), when the crack position is constant, the natural frequency of a simply su-pported beam is inversely proportional to the crack depth. In Fig. 4(b), in the case of ξ_c =0.5 the natural frequency ratios of the second mode of vibration are unit due to the mode shape of the beam. In addition, the natural frequency of a simply supported beam is decreased rapidly when the crack depth H is larger than 0.5.

Figure 5 represents the frequency ratio of a simply supported beam according to the crack position. When the crack position exists in the center of a simply supported beam, the difference of frequency ratio of a cracked beam in the two cases of H =0.1 and H =0.3 is about 1.6%. And the difference of frequency ratio of a cracked beam in the two cases of H =0.3 and H =0.5 is about 6.3 %.

The reduction of natural frequency of a cracked beam for the first and second modes of vibration is shown in Fig. 6. As shown in these figures, the reduction of natural frequency of a simply supported beam is related to the depth and the position of a crack, and the mode shapes.

Table 3 shows the natural frequency of a simply supported beam with the double cracks for the first mode of vibration. In addition, comparison between theoretical results and experimental measurements is also given. In these results, the maximum difference between the two results is less than 2.06 %. It can be found that that the theoretical results are a good agreement with the experimental measurements. Table 4 represents the natural frequency of a double-cracked beam according to the position and depth of each crack.

Figure 7 shows natural frequency of a simply supported double-cracked beam according to the crack depth and crack position of each crack for the first mode. Totally, when the crack positions are constant, as the depth of cracks is increased, the natu-

Table 3. Natural frequency of a simply supported beam with the double cracks for the first mode(ξ_{c2} =0.5, H_1=0.5).

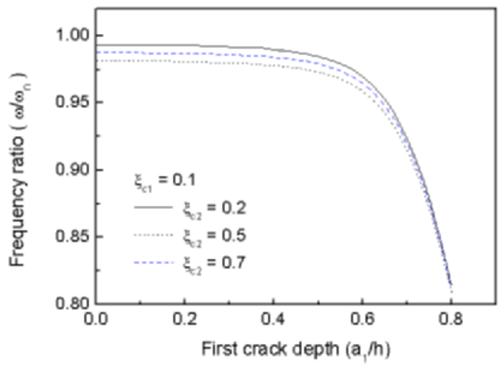
First crack position: ξ_{c1}	Second rack depth : H_2	Natural frequency [Hz] T:Theory	Natural frequency [Hz] E:Experiment	Error(%) :|E-T|/E×100
0.1	0.1	148.96	149.5	0.36
	0.3	146.56	146.8	0.16
	0.5	137.30	139.8	1.79
0.3	0.1	141.89	142.0	0.08
	0.3	139.81	140.8	0.70
	0.5	131.72	134.5	2.06

Table 4. Natural frequency of a simply supported beam with the double cracks for the first mode.

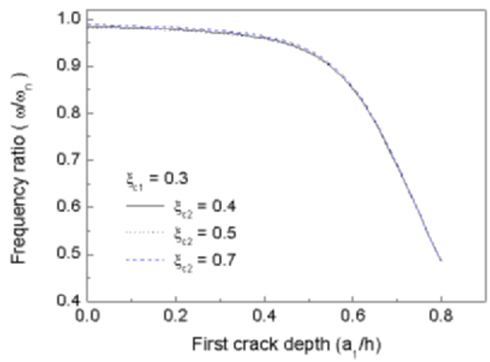
First crack position $\xi_{c1}=x_{c1}/L$	Second crack position $\xi_{c2}=x_{c2}/L$	$H_1(a_1/h)=0.5$			$H_2\ (a_2/h)=0.5$	
		$H_2=0.1$	$H_2=0.3$	$H_2=0.5$	$H_1=0.1$	$H_1=0.3$
0.1	0.2	149.16	148.31	144.71	145.92	145.69
	0.5	148.96	146.56	137.30	138.29	138.11
	0.7	149.07	147.48	141.07	142.13	141.93
0.3	0.4	141.91	140.01	132.52	139.17	145.69
	0.5	141.89	139.81	131.72	138.16	138.11
	0.7	141.98	140.63	135.11	141.98	140.63
0.5	0.6	138.09	136.35	129.44	139.08	137.12
	0.7	138.16	136.89	131.72	141.89	139.81
	0.8	138.23	137.56	134.71	145.66	143.42

ral frequencies of a simply supported double-cracked beam are decreased. Specially, when the first crack position exists in the center of a simply su-pported double-cracked beam, the natural frequencies of a double-cracked beam are most sensitive to the crack position.

Figures 8 and 9 show the natural frequency of a simply supported double-cracked beam according to the crack depth. In this case, the double crack are located at ξ_{c1}= 0.3 and ξ_{c2}= 0.6 from the left-hand hinged end of the beam, respectively. In these results, we will predict easily the natural frequencies of a simply supported double-cracked beam according to crack depth. That is, the change in natural frequencies can be used to estimate the depth of the cracks. Inversely, the change in the depth of the cracks can be used to estimate the natural frequency of a simply supported double-cracked beam. In figures, the change of the natural frequency is plotted versus the depth(H_2) of the second crack for four different values



(a) ξ_{c1}=0.1



(b) ξ_{c1}=0.3

Fig. 7. Natural frequency of a double-cracked beam according to the crack depth and crack position for the first mode.

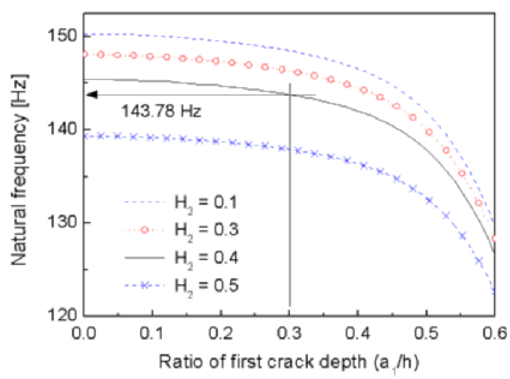


Fig. 8. Natural frequency of a double-cracked beam versus depth of the first crack (ξ_{c1}= 0.3, ξ_{c2}= 0.6).

values of the depth(H_1) of the first crack. Using the measured natural frequency of the cracked beam, a horizontal line can be drawn. The intersection of this line with the (H_1) curves represents the depth of the

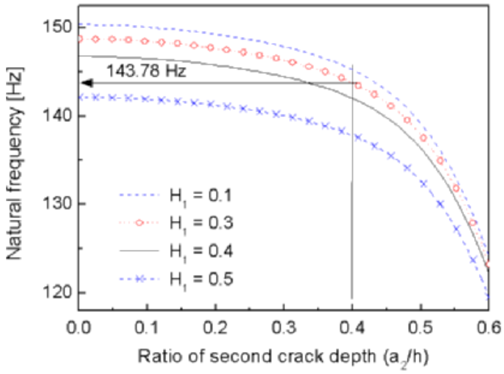


Fig. 9. Natural frequency of a double-cracked beam versus depth of the second crack(ξ_{c1} = 0.3, ξ_{c2} = 0.6).

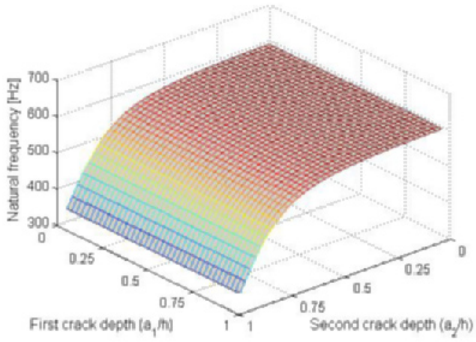
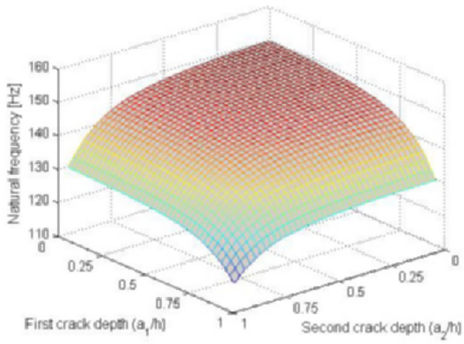
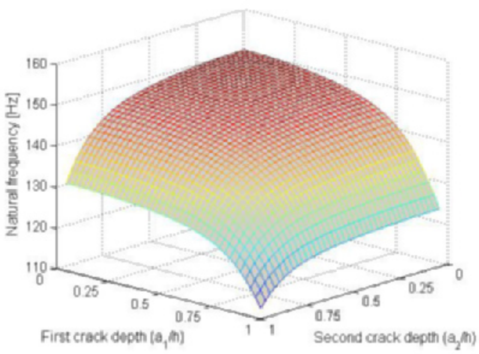


Fig. 11 Contours of natural frequency of a double-cracked beam due to the crack depth for the second mode (ξ_{c1} = 0.5, ξ_{c2} = 0.7)

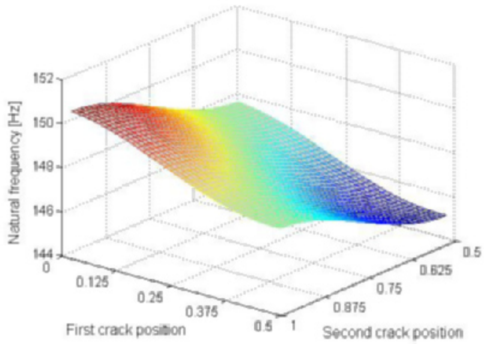


(a) ξ_{c1} = 0.3, ξ_{c2} =0.7

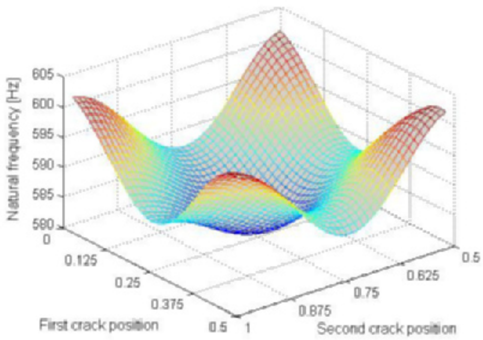


(b) ξ_{c1} = 0.5, ξ_{c2} = 0.7

Fig. 10. Contours of natural frequency of a double-cracked beam due to the crack depth for the first mode.



(a) First mode



(b) Second mode

Fig. 12. Contours of natural frequency of a double-cracked beam due to the crack position (H_1= 0.3, H_2= 0.3)

first crack, while the ordinate of the intersection point represents the depth of the second crack. In our example, we can estimate that the natural frequency of a simply supported double-cracked beam is about 143.78[Hz] when the depths of the crack are H_1=0.3 and H_2= 0.4.

Figures 10~12 show the changes of the natural frequencies of a simply supported double-cracked beam due to the crack positions and the depth of the cracks. Figure 10 and Fig. 11 are the first vibration mode and second vibration mode, respectively. Figure 12 represents the contours of natural frequency of a double-cracked beam due to the crack position for $H_1 = H_2 = 0.3$. In these figures, the position of the cracks from the left-hand supported end gradually moves to the right-hand supported end of the beam with increasing of the frequencies of a simply supported beam. When the crack positions are constant, the natural frequencies of a simply supported double-cracked beam are in inverse proportion to the crack depth.

5. Conclusions

In this paper, the influences of the crack depth and the crack position were studied on the dynamic behavior of a simply supported double-cracked beam by the numerical method and the experimental measurements. A simply supported beam is modeled by the Euler-Bernoulli beam theory. The equation of motion is derived by using Hamilton's principle. The double-cracked beam has been treated as three undamaged segments connected by a rotational elastic spring at the each cracked section. The stiffness of the spring depends on the crack depth and the geometry of the cracked section. The main results of this study are summarized as follows:

The effects of the position and depth of each crack on the natural frequency of a simply supported double-cracked beam was investigated theoretically and experimentally. It was shown that, when the crack positions are constant, the natural frequencies of a simply supported double-cracked beam are inversely proportional to the crack depth and when the first crack position exists in the center of a simply supported double-cracked beam, the natural frequencies of a double-cracked beam are most sensitive to the crack position.

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